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Socio-Demography and Income Inequality

– An Overview

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Zusammenfassung*


Summary*

This discussion paper deals with connections between the variables (socio-)demography and income inequality (in a cross-sectional perspective against the background of Germany’s ageing process). Since the factors of influence and their relationships to each other are extremely complex, multifactorial approaches for explaining the distribution of personal incomes seem to be obvious. In this context the discussion paper focuses on the role of (socio-)demography in explaining the measured income inequality insofar as fundamental mechanisms of action and transmission channels are main subjects of discussion. This contains an overview over former analyses concerning the questions important in this paper.

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Content

1. Introduction 5

2. Multifactorial explanation attempts of the personal income distribution 7
   2.1 Grüske’s multifactorial approach 7
   2.2 Von Weizsäcker’s multifactorial approach 9

3. Demographic factors of influence concerning the personal income distribution 13
   3.1 Population’s growth and income inequality: basic connections 13
      3.1.1 Population size and income inequality 13
      3.1.2 Differential fertility and differential mortality 15
   3.2 Direct and indirect demographic impacts on the inequality of equivalent incomes 17
      3.2.1 Preliminary remark 17
      3.2.2 The direct demographic influence of the household structure 18
      3.2.3 Labour market 20
      3.2.4 Capital market 23
      3.2.5 Tax-transfer system 25
      3.2.6 Application of incomes 26
      3.2.7 Macroeconomic level 26
   3.3 A model embracing the relations between socio-demography and income inequality 27

4. Shift-share and decomposition analyses concerning the connection between demography and income inequality 32

5. Concluding remarks 40

References 41
1. Introduction

This paper deals with connections between socio-demography and income inequality. Socio-demographic variables are e. g. age, sex, employment status, household size, household composition, family status, religion, migration background, or status of education.

Against the background of Germany’s ageing process, the social relations between generations – trenchantly: between the “old” and the “young” – become increasingly important. On the founded supposition that individual possibilities for development are primarily based on economic scopes for action, a comprehensive well-being analysis seems to be adequate.¹ Ideally, such a well-being analysis comprises, beyond the typical indicator income, further well-being indicators like wealth or private consumption.²

The ageing of societies primarily concerns indirect demographic effects which are, not least, mediated by the tax-transfer system. At this, in Germany especially the pay-as-you-go-procedure of the national retirement system and its intergenerational distributional problems are focused. In a politico-economic view the demographic change also causes – on the basis of (utility-)maximising decisions of governments – modified political decisions. Furthermore, the demographic development affects the accrual of incomes (via changes in relative prices and via disincentives concerning the beginning of work) as well as the application of incomes (via age-specific consumption and saving rates or via age-differentiated consumption structures). In this context (at least partly), the aspects of the distribution, the accrual, and the application of incomes are connected interdependently. Thus, the question about the distribution of income is linked up with aspects of economic growth. This underlines the relevance of connections between (socio-)demography and personal income distribution.

Since social conflicts reveal themselves on a cross-sectional level, the consideration of cross-sectional income inequality is highly important in a socio-political sense (not least for the practical social policy). This comprises the determination of the personal (periodical) income distribution as well as the investigation of connections between (cross-sectional) period incomes and (longitudinal) lifetime incomes. The comparative analysis of the distribution of lifetime incomes makes sense, inter alia because such an analysis can avoid contortions which are elicited by population’s age structure. Performing such an analysis, cohort effects can be isolated. Nevertheless, this paper concentrates on cross-sectional aspects of the personal income distribution.

Still until the 1980s, the connection between demography and income distribution was treated as an orphan. Primarily, this had two causes: firstly, the connection between population and production seemed to be more relevant, and secondly, the direction between demography and income distribution seemed to be clear in the sense that population’s growth would cause a more unequal income distribution. This causality returned to the classics (with their emphasis of the functional income distribution): the classics argued that an increase of the landowners’ income and a decrease of the workers’ income would be inevitable. In their opinion the increase of the landowners’ income would result from the principle of differential rents, i. e. that the profit stemming from landownership would be determined through the yield of the worst ground; the latter was called the marginal yield. In case of population’s growth the fertile ground would be scarce because of increasing needs for food. This would increase the share of rents with respect to national income. Furthermore, a rapidly growing population would diminish the capital intensity, and because of this the relative price of capital would rise. How far these effects do influence income distribution, depends on the concrete degree of substitution of capital through labour.³

¹ Beneath monetary incomes and charges, non-monetary income elements and price privileges are relevant (see Fachinger and Schmähl 2004, p. 537).
² These indicators might be pooled in a single indicator (e. g. following the concept of a well-being indicator proposed by Weisbrod and Hansen 1968).
Even in case that a compensation of the price increases of capital by the substitution of capital through labour takes place, there are some reasons for an increasing income inequality as a consequence of population’s growth.4

1. A rising fertility in the lower than in the higher income ranges leads in case of relatively low individual opportunities of advancement to an increasing income inequality.

2. High birth rates probably reduce the expenditures for education per child; this generates a higher income inequality in the next generation.

3. Rising numbers of inhabitants typically increase the population share of the young age groups which, by tendency, are located at the bottom ends of jobs’ and incomes’ hierarchies; this corresponds with increasing income differences.

Paglin (1975) and Kuznets (1976) opened a corresponding, international debate in the mid-1970s by asking how demographic variables do affect the measured (cross-sectional) income inequality.5 In their opinion a positive statistical correlation between household size and household income would be realistic, and the household incomes would depend on the age of the household heads in an inverse U-shape. Taking together both connections, this formulates dependencies between the household incomes’ distribution and the distribution of households concerning their size and concerning the age distribution (of household heads).6 Particularly three pure demographic factors can be differentiated with respect to the age distribution: the mean age (of the labour force), the dispersion (of the labour force), and the old-age dependency ratio.7

Beneath the age distribution, e. g. the changes in household composition, which are caused by social change, are very relevant for inequality purposes. Generally speaking, in the context of decomposing inequality, differentiations between the effects of different kinds of income and different income structures are as important as socio-demographic influences (e. g. age or household structures).

Since the factors of influence on the personal income distribution and on each of their internal relationships are highly complex, multifactorial approaches for explaining aspects of the personal income distribution appear to be obvious. Questions, arising in this context, are: How does the tax-transfer system affect the secondary income distribution via redistributions of market incomes? What kind of influence on the personal income distribution do individual preferences or decisions of entrepreneurs concerning the employment of young versus old persons have? Are changes of household size or household composition very relevant for aspects of the personal income distribution? Chapter 2 is concerned with such questions. Subsequently, Chapter 3 focuses insofar on the role of (socio-)demography in explaining the measured income inequality as fundamental mechanisms of action and transmission channels between (socio-)demography and income inequality are addressed. Based on these considerations, Chapter 4 gives an overview over former investigations concerning the issues discussed in this paper. Last but not least, in Chapter 5 the discussion paper’s considerations and findings are prospectively summarised.

4 See Felderer and Sauga 1988, p. 208. In an empirical cross-country study on a cross-sectional database Chenery et al. 1974, p. 17, found that an increase of the population’s rate of growth in the amount of one percent leaded to an increase of income inequality in the amount of 1.6 percent.


7 See von Weizsäcker 1993, pp. 23-25.
2. Multifactorial explanation attempts of the personal income distribution

Since singular explanation attempts – like the human-capital approach – can only explain a small part of the whole income variance\(^8\) or are economically unsatisfiable – like the (pure) stochastic approaches\(^9\) –, it seems justified to recur on multifactorial explanatory models. For example, Jenkins' (1995) analysis, which will be considered in more detail in Chapter 4, differentiates between the following explanatory factors concerning the distribution of personal household net equivalent incomes:\(^10\)

1. changes in age structure,
2. changes in household composition,
3. changes in employment structure,
4. changes in the structure of branches,
5. changes with respect to unemployment,
6. changes of the business cycle,
7. changes in the tax and in the transfer system,
8. changes in inequality of wages, and
9. changes in capital incomes.

Subsequently, two older but still important multifactorial approaches will be considered: Grüske's (1985) and von Weizsäcker's (1986) approach. The aim in this context is to give an impression of the diversity of the personal income distribution concerning its causes.

2.1 Grüske's multifactorial approach

Grüske (1985) named individual explanatory factors as well as market-determined and institutional factors.

The *individual* factors were organized by Grüske in a tripartite classification:

1. non-influenceable individual factors:
   a. inborn characteristics (sex, etc.),
   b. environment (parents, cultural environment),
   c. age,
   d. inherited wealth;

2. partly influenceable individual factors:
   a. health,
   b. abilities,
   c. dynamic factors (motivation, mobility, diligence, personality, etc.),
   d. "connections";

---

\(^8\) Mincer (1976, p. 146) e. g. – one of the apologists of the human-capital approach – calculated determination coefficients only in the amount of about 50 percent at the maximum for the regression between labour income and the main explanatory variable education. Other authors estimated still lower explanatory values for the human-capital approach. Thurow (1981, p. 167) reported for this connection determination coefficients between only 20 and 30 percent. Concerning the human-capital model and its explanatory power see also Wolff 2009, pp. 247-258.

\(^9\) For an overview over such approaches see e. g. Sahota 1978. Blinder (1974, p. 7) encapsulates the critique of the (pure) stochastic approaches: "Assuming a stochastic mechanism, no matter how complex, to be the sole determinant of income inequality is to give up before one starts. It is antithetical to the mainstream of economic theory which seeks to explain complex phenomena as the end result of deliberate choices by decision-makers. (…) One would hope that economics could do better than that."

\(^10\) See Jenkins 1995, pp. 32-35. In this context see also Wolff 2009, Part II, for a similar methodological perspective.
3. individually influenceable factors:
   a. (further) education,
   b. job, job experience, training on the job,
   c. individual utility functions with decisions e.g. concerning working versus leisure time or consumption versus saving,
   d. formation of households.

Out of these individual factors only individual wealth influences income directly in Grüske’s model. Wealth in period t is defined as follows (on the simplifying supposition of constant saving rates and constant rates of return over time):

$$(1) \quad V_t = q \cdot (1 + q \cdot s)^{1 - \tau} \cdot V_0^t + \sum_{i=1}^{t-1} s \cdot (1 + q \cdot s)^{t-1-i} \cdot Y_i.$$

In this equation the meaning of the variables is: t: period of time (of occupation), $V_0$: an individual’s wealth inherited in period $\tau \geq (t=1)$ or possessed at the beginning of occupation ($\tau = 0$), s: an individual’s saving rate, q: an individual’s rate of return on financial and non-financial assets, Y: individual disposable income.

With respect to Equation (1), a positive correlation between saving rate and inherited wealth seems plausible; the same holds true for the connection between saving rate and disposable income. A person with a high income or wealth will (can) normally form more savings than a person with a low income or wealth. The multiplicative conjunction of these variables in Equation (1) leads to a right-skewed wealth distribution. Additionally, this right-skewness is still reinforced via the multiplicative linkage and via the positive correlation of saving rate, income, and wealth with the rate of return on assets.

For almost all other individual factors Grüske also considered a right-skewed distribution (with corresponding effects on the overall personal income distribution). In distinction from wealth the other individual factors influence income only indirectly – as determinants of labour supply.

In order to determine the personal (labour) income distribution it is necessary to consider the labour demand as well as the labour supply. In this context Grüske assumed (empirically well-founded) that the individual labour income increases with a rising qualification level; here, the qualification level is represented by education. On the further supposition of a right-skewed distribution of the several qualification groups, even an equilibrium on the labour market (labour supply = labour demand) generates a right-skewed (labour) income distribution. Because of the plausible positive correlation between qualification level and (labour) income the (density) curve of the personal (labour) income distribution is more right-skewed than the frequency curves of the several qualification levels.

Furthermore, in his model Grüske considered effects resulting from disequilibriums on labour market. Exemplarily, he assumed that in the lower area of qualifications the supply is higher than the demand for labour (et vice versa in case of high qualifications). Such disequilibriums lead to diminishing (labour) incomes of the lower qualification levels and to rising (labour) incomes of the higher qualification levels. This causes a strengthening of the right-skewness of the (labour) income distribution compared with the situation of an equilibrium on labour market.

Grüske considered the institutional factors of the personal income distribution only rudimentarily. He stressed the point that the so-called hierarchy effect, which possibly holds true for the upper income area, should be complemented with the influence of collective bargainings for middle and bottom income groups. The hierarchy effect means, simply speaking, that the...
payments of managers are positively correlated with the size of their staff. The role of tradition should also be regarded as an institutional factor – in the sense that it can influence the hierarchy effect as well as the results of collective bargainings.

Last but not least, Grüske characterised chance as a principally explainable residuum, and he stressed the point that stochastic influences can affect the other variables of the model permanently as well as transitorily.

The following figure gives an overview over Grüske’s approach; basically, this model is – in my eyes – helpful for structuring the coherences of the personal income distribution and its determinants.

Figure 1: Grüske’s model

Source: Grüske 1985, p. 51

2.2 Von Weizsäcker’s multifactorial approach

Another multifactorial approach stems from von Weizsäcker (1986). It solely deals with the distribution of labour incomes. Loosely speaking, it is a synthesis between pure stochastic and human-capital theoretical approaches (but additionally taking into account further explanatory variables). With respect to the extent of the object of study and to the number of explanatory variables von Weizsäcker’s approach is not as comprehensive as Grüske’s model sketched in Section 2.1 (because of a more restrictive income definition and because of fewer explanatory factors), but it is theoretically, i.e. from a microeconomic point of view, much more elaborated than Grüske’s model.

11 See Lydall 1968, pp. 128-129.
Von Weizsäcker divided that part of the income equation which represents the proportional income effect in the usual stochastic models\(^{12}\) – i. e. \((1+\varepsilon_t)\) – in an expected factor of growth of the individual income \((1+T_t)\) and in a residual factor of growth that is determined purely stochastic \([(1+e_t)]\).

With \((1+\varepsilon_t) := (1+T_t)(1+e_t)\) the individual labour income in period \((t+1)\) equals:

\[
Y_{t+1} = Y_t \cdot (1 + T_{t+1}) \cdot (1 + e_{t+1})
\]

or

\[
Y_{t+1} = Y_0 \cdot \prod_{i=0}^{t} (1 + T_{i+1}) \cdot \prod_{i=0}^{t} (1 + e_{i+1}).
\]

Alternatively, the expected labour income in period \((t+1)\) can be considered. On the suppositions that \(E(e_{t+1}) = 0\) and hence \(E(1+e_{t+1}) = 1\), the expected labour income in \((t+1)\) is:

\[
A_{t+1} = E(Y_{t+1})
= E(Y_t) \cdot (1 + T_{t+1})
= Y_0 \cdot \prod_{i=0}^{t} (1 + T_{i+1}).
\]

Inserting Equation (4) into Equation (3) yields:

\[
Y_t = A_t \cdot \prod_{i=0}^{t} (1 + e_i)
\]

or (using logarithms: \(X_t := \log Y_t\) and \(u_i := \log (1+e_i)\))

\[
X_t = \log A_t + \sum_{i=0}^{t} u_i.
\]

On the supposition that the \(u_i\) are independently and identically distributed and on the basis of the central limit theorem, for large values of \(t\) an asymptotical normal distribution of the logarithmical labour incomes \(X_t\) or – equivalently: – an asymptotical log-normal distribution of the labour incomes \(Y_t\) results.

The above mentioned income component \(A_t\) can – referring to Friedman’s permanent income hypothesis – be named as a permanent income component, and \(C_t := \prod_{i=0}^{t} (1 + e_i)\) can be labeled as a transitory income element.

\(^{12}\) The law of the proportional effect (the so-called "loi de l’effet proportionnel") was firstly (and explicitly) formulated by Gibrat. With respect to the variable income it means that the relative rate of growth concerning income is a random variable and that the probability distribution of the proportional income changes is independent of the current income level. The individual income in period \(t+1\) results from the individual income in the previous period \(t\), and this is accompanied by a random impact. The random impact is a multiplicative one (see Gibrat 1931, pp. 63-64).
The transitory character of $C_t$ reveals itself by the fact that this variable is individually not ex-
pectable. The shocks represented by $C_t$ change their values continuously. Because of this, in
the model it is assumed that the expectancy value of $C_t$ equals one (resulting from $E(e_t) = 0$);
i. e., the income effects of the stochastic shocks cancel each other in a long-term perspec-
tive, and they do not have a permanent impact on the income levels individually expected.

In order to switch from the individual labour income distribution to the whole economy’s la-
bour income distribution, inter alia it must be considered that (in von Weizsäcker’s model) the
several individuals differ from each other concerning their expected income levels $A_t$.

Assuming stochastic independence between $A_t$ and $C_t$, the theoretical distribution function for
labour income in $t$ is:

$$ F_{Y_t} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{A_t}(a_t) \cdot f_{C_t}\left(\frac{y_t}{a_t}\right) \cdot da_t \cdot dy_t. \tag{7} $$

[Remark: whereas $f_{A_t}$ and $f_{C_t}$ represent the density functions of $A_t$ and $C_t$, $y_t$, $a_t$, and $(y_t/a_t)$ symbolise the realisations of the random variables $Y_t$, $A_t$, and $C_t$.]

Since $C_t$ solely reflects stochastic impacts (see above), an economisation of the model is
realised via $A_t$. Because of the foundation of von Weizsäcker’s model on human-capital theo-
ry an individual possesses a certain quantity of human capital ($HK_t$) in period $t$. It is assumed
that this individual only takes the fraction $(1-h_t)$ for activities concerning the acquisition of
money; the residual fraction $h_t$ he uses – during his working period – for investments in (fur-
ther) education.

Following these premises, the (expected) disposable labour income in $t$ ($= A_t$) consists of the
human-capital stock in $t$ valued by the factor price $R$ ($= R \cdot HK_t$). From this, the income lost
as a consequence of fees for educational purposes ($= R \cdot h_t \cdot HK_t$) and the quantity of goods
and services purchased for education and valued by a price ($= P \cdot D_t$) need to be subtracted:

$$ A_t = R \cdot HK_t - R \cdot h_t \cdot HK_t - P \cdot D_t $$

$$ = R \cdot HK_t \cdot (1-h_t) - P \cdot D_t $$

with: $h_t \in [0; 1]$, $D_t \in [0; \infty]$, $R > 0$, $P > 0$ ($t = 0, 1, 2, \ldots, n$).

The investments in (further) education imply that the individual human-capital stock is varia-
ble over time. The new production of human capital in $t$ represents “learning by doing” during
the working period – i. e. during $(1-h_t)$ – as well as the explicit accumulation of human capital
in the stadium of education – i. e. during $h_t$.

In this context, the accumulation of human capital can be described via a production function
$Q_t$, e. g. via a Cobb-Douglas production function with decreasing economies of scale:

$$ Q_t = b_0 \cdot (h_t \cdot HK_t)^{b_1} \cdot D_t^{b_2} $$

with: $b_0 > 0$, $b_1 > 0$, $b_2 > 0$, $b_1 + b_2 < 1$ ($t = 0, 1, 2, \ldots, n$).

In Equation (9) the elasticities of production $b_1$ and $b_2$ of the inputs $h_t \cdot HK_t$ and $D_t$ shall have
the same amount for all members of society. In contrast, $b_0$ shall vary individually: $b_0$ symbol-
ises an individual’s ability to increase his own production capacities during the working peri-
od.
In von Weizsäcker’s model \( b_0 \) is influenced by a number of factors. Direct factors of influence are the individual learning’s ability (LA), Lydall’s D factor (DF)\(^\text{13}\) and some other characteristics of personality (QPC; the willingness of leading and managing, the ability or willingness to accept responsibility, etc.), so-called “class-rank” variables (CR; e. g. school marks), and the quality of schooling (SQ). Indirectly \( b_0 \) is influenced – via LA and partly via DF – by hereditary factors (G), family background (HO), and cultural impacts (CU). Thus, for \( b_0 \) the following function results:

\[
\begin{align*}
 b_0 &= b_0 \left( LA(G, HO, CU); DF(HO, CU); QPC; CR; SQ \right) \\
\text{with: } &\frac{\partial b_0}{\partial LA} > 0, \frac{\partial b_0}{\partial DF} > 0, \frac{\partial b_0}{\partial QPC} > 0, \frac{\partial b_0}{\partial CR} > 0, \frac{\partial b_0}{\partial SQ} > 0.
\end{align*}
\]

Beyond that, it must be considered that a certain quantity of human capital needs to be reduced in value per period (\( := \Omega \)) e. g. because of deteriorations of the short-term memory with increasing age or because of devaluations of qualification in the course of technological changes.

Considering \( \Omega \), leads to the human-capital stock in period \((t+1):\)

\[
\begin{align*}
 HK_{t+1} &= HK_t + Q_t + c \cdot (I - h_t) \cdot HK_t - \Omega \cdot HK_t \\
\text{with: } HK_0 &> 0 \text{ given and } t = 0, 1, 2, \ldots, n-1.
\end{align*}
\]

In von Weizsäcker’s model an individual is maximising his labour income over his entire life-cycle. That means that an individual tries to maximise the present value of his disposable labour incomes per period – i. e. u. The interest rate on the (perfect) capital market \( r (> 0) \) represents the discounting factor:

\[
\begin{align*}
 v_t := \sum_{t=0}^{n} A_t \cdot (I + r)^{-t} \rightarrow \max.!
\end{align*}
\]

Equation (11) for \( HK_{t+1} \) is the restriction within this problem of maximisation.

In the context of von Weizsäcker’s approach it can be dealt with inequality differences between longitudinal lifetime incomes on the one hand and cross-sectional period incomes on the other hand. By tendency, the period incomes are at least on a level with the lifetime incomes (the latter in the sense of the present value of the individual period incomes), the

- larger the sum of fertility and mortality rate,
- larger the population shares of the younger cohorts,
- smaller the interest rate \( r \),
- worse the abilities for learning by doing,
- faster the decreases of productivity caused by ageing,
- smaller the human-capital elasticities of production \( b_1 \) and \( b_2 \),
- larger the mean efficiency of full-time schooling \( \mu_{a1} \),
- larger the average initial human-capital stock \( \mu_{a0} \)

is/are.\(^\text{14}\)

An increasing rate of ageing \( \Omega \) leads, e. g. and ceteris paribus, to a reduction of inequality of lifetime incomes. The reason for this is that to a greater degree more “able” individuals (i. e. individuals with a higher \( a_1 \) and/or a higher \( b_0 \)) are involved in human-capital losses com-

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\(^{13}\) Lydall’s D factor comprises non-cognitive abilities like personality characterised among themselves by a number of elements, e. g.: assertiveness, tenacity, ambition, willingness to work hard, willingness for risk taking, or ability to cooperate with other people (see Lydall 1981, p. 135).

\(^{14}\) Remark: the inequality indicator used by von Weizsäcker is the squared coefficient of variation.
pared with less “able” persons. The same holds true e. g. for a deterioration of the abilities for learning by doing.

Von Weizsäcker’s approach comprises a number of empirically testable hypotheses which can be allocated to the groups of direct versus indirect (socio-)demographic influences. In the following, we will deal with such influences and with their connections to the personal income distribution in more detail.

3. Demographic factors of influence concerning the personal income distribution

Out of the factors of influence sketched in Section 2.1, in this chapter we will refer to the individual factors (age, sex, education, etc). In this context an impact from demography towards income inequality will be assumed. Influences, which are contrary to this – i. e. impacts from income inequality towards demography –, will be neglected.

3.1 Population’s growth and income inequality: basic connections

In this section “pure” effects generated by population size (Section 3.1.1) as well as differential population size effects (Section 3.1.2) will be treated. It must be stressed that the corresponding remarks will concentrate on individual incomes, i. e. that the household context will be neglected; the same holds true for equivalence scales. Such necessary methodological extensions with respect to the measurement of income inequality will be introduced not until Section 3.2.

3.1.1 Population size and income inequality

Concerning the following figure it is assumed that the number of births increases continuously starting in a certain point of time. For example, this means a doubling of births, so that the number of persons one year old will be twice as high in t+1 than in t, the same shall be valid for persons two years old, etc. From the age of 20 years on, persons shall be part of statistics of (income) distribution. Thus, (ceteris paribus) it will last 20 years until the model’s population will raise its size (by doubling the size of the youngest cohort, consisting of persons 20 years old). Assuming that all people will die in the age of 80 years, it will last exactly 80 periods until the persons born in the starting period will have been completely involved in the demographic changes sketched, and a new stationary state will be reached. This new stationary state is characterised by twice as much people in each age cohort compared with period t (which means a doubling of the model’s population). Moreover, this kind of purely concentrating on changes of population size produces the same (relative) age distribution in the new stationary state (in t+80) as in the initial situation (in t).15

15 See Pohmer 1989, p. 105.
Hence (ignoring cohort effects), the income inequality in the last period (in t+80) is exactly as high as in the initial period (in t). Compared with this, in the transition period between the two stationary states changes in the measured inequality exist. Decomposing total income inequality into a within-group and a between-group component, principally reveals that the measured income inequality (ceteris paribus) the higher is the larger a) income inequality in each age group is and b) deviations of the age-specific mean incomes from the social mean value are. Concerning aspect a) it is an empirical finding that a U-shape of the internal income inequality over the several age groups typically occurs. The age-specific mean income (aspect b)) has an inverse U-shape in many empirical cases; i. e., in young and in old years of age the mean incomes are lower (and in the middle stage of life they are higher) than the social mean income.\(^{16}\)

These processes and a “younger” age structure lead to an increase of income inequality for 20 periods. Subsequently, inequality decreases (in Figure 2 from the 40\(^{th}\) period on) because the frequencies of the age groups with a relatively low internal income inequality and an income level near the social mean income increase. At the end of the demographic transition the frequencies of the older persons with a relatively high group-specific income inequality and a mean income below the overall mean income rise, and so again the income inequality increases (starting with the 60\(^{th}\) period). When (in the 80\(^{th}\) period) the demographic processes are terminated, total income inequality will reach its initial level, as mentioned above.\(^{17}\)

In the inverse case of continuously decreasing frequencies in the several age classes, the inequality processes will be quasi mirror-inverted, as is shown in Figure 3. At first the inequality decreases in order to increase later at a certain period of time (in the 40\(^{th}\) period), and then (from the 60\(^{th}\) period on) it decreases towards the initial level which is reached at the end of the relevant process (in the 80\(^{th}\) period).\(^{18}\)

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\(^{16}\) Below we will deal with this issue in more detail.

\(^{17}\) See Pohmer 1989, pp. 106-108.

\(^{18}\) See Pohmer 1989, pp. 111-114.
Early analytical models concerning the (interdependent) connections between income distribution and population’s growth stem from Ogawa (1978), Winegarden (1978), and Repetto (1979). Winegarden stated, on the basis of a simultaneous multi-equations regression, that empirically a deceleration of population’s growth would have an inequality reducing effect. Furthermore, he explained that a smaller degree of income inequality would dampen population’s growth. Especially it was found that the impact of population’s growth on income distribution would have been (much) stronger than conversely. Later studies, e. g. done by Rodgers (1983) or Winegarden (1980, 1985), used time-lags for the connections between income inequality and demography – modeling the demographic variables as lagged variables; the resulting models were (partly) recursive.19

3.1.2 Differential fertility and differential mortality

For a sustainable population’s ageing a persistent reduction of the growth rates of births is necessary. In this context, differential fertility, which e. g. means a reduction of the fertility of the upper income groups, has other consequences for age structure and income inequality than a general fertility reduction across all income classes.20

Lam (1986) simulated several inequality processes as reactions on differential fertility (by using the alternative inequality indicators logarithmic income variance and squared coeffi-

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19 See Heerink 1994, p. 5.
20 Furthermore, migration does not influence per se but through (age-specific) changes in the immigration rates population’s ageing and income inequality. See von Weizsäcker 1996, p. 3.
cient of variation, and by applying cross-section data of Brazil).\textsuperscript{21} Differential fertility in Lam’s study also meant a faster population’s growth of the bottom income groups compared with the upper income classes. Whereas in this case the effects on the arithmetic mean of the personal income distribution were unambiguous (in the direction of a decreasing mean value), the corresponding inequality effects were less clear. Lam carved out the following conditions for constant or changing values of the logarithmic variance and of the squared coefficient of variation:\textsuperscript{22}

(13) **Logarithmic variance (LV):**

$$LV_{n+m} \begin{cases} > & LV_n, \\ < & \end{cases}$$

if: $$m' = \begin{cases} > & n \cdot (\mu_m - \mu_n)^2 - n \\ < & \frac{LV_n - LV_m}{n} \end{cases}$$

and

(14) **squared coefficient of variation (QCV):**

$$QCV_{n+m} \begin{cases} > & QCV_n, \\ < & \end{cases}$$

if: $$m' = \begin{cases} > & n \cdot \left( \bar{Y}_m^2 \cdot QCV_m + \bar{Y}_n^2 \cdot QCV_n - 2 \cdot \bar{Y}_m \cdot \bar{Y}_n \cdot QCV_n \right) + n \cdot \left( \bar{Y}_m - \bar{Y}_n \right)^2 \\ < & \frac{QCV_n - QCV_m}{\bar{Y}_m^2} \end{cases}$$

[LV: logarithmic variance, n: initial population size, m: “added” population, \(\mu_m\): logarithm of the arithmetic mean income of population m, \(\mu_n\): logarithm of the arithmetic mean income of population n, QCV: squared coefficient of variation, \(\bar{Y}_m\): arithmetic mean income of group m, \(\bar{Y}_n\): arithmetic mean income of group n].

According to Lam, income inequality will “undoubtedly” increase as a consequence of differential fertility if the (“mover’s”) probability for a child in the lowest income class to climb into a higher income class is lower than the (“stayer’s”) probability to stay in the lowest income class.\textsuperscript{23}

Ahluwalia (1976) also illuminated connections between demography – more specific: population’s growth – and income inequality in a differential perspective. He assumed no mobility between high and low incomes. Additionally, he postulated – like Lam – that the poor households would have a faster population’s growth than the richer households. In case of a large population’s growth this would result in a remarkable increase of income inequality.\textsuperscript{24}

\textsuperscript{21} The corresponding considerations are based on individual incomes. With a few modifications they can be transferred to the level of household (equivalent) incomes relatively easily.

\textsuperscript{22} See Lam 1986, pp. 1104-1106.

\textsuperscript{23} See Lam 1986, p. 1110. Chu 1987 showed in a replica to Lam’s article that the rule mentioned is a necessary but not a sufficient criterion in this context.

\textsuperscript{24} See Ahluwalia 1976, pp. 326-327; for a similar approach see Morley 1981.
Like both aforementioned studies, Repetto (1978) dealt with the interactions between fertility and income inequality. He stated a non-linear, typically U-shaped connection between income and fertility. On this basis, Repetto noted with respect to the reverse impact of differential fertility on income inequality that a low fertility at the bottom and at the top of income’s hierarchy would tend to an increase of the labour income inequality.

Concerning differential mortality, Grimm and Cogneau (2005) showed that a negative as well as a positive correlation between mortality and income tend to generate inequality reductions. The reason for this would be that in both (correlation) cases individuals at the bottom or at the top of the income distribution would be “eliminated”. In distinction from that (“truncated”) case, in Grimm and Cogneau’s model a very high mortality in the middle income classes leads to an inequality increase.

Whereas – in a very simplified perspective – differential fertility (at the bottom and/or at the top of income’s hierarchy) “adds” persons (or households) at the margins of the income distribution with the consequence of an increasing income inequality by tendency, differential mortality (at the bottom and/or at the top of income’s hierarchy) “eliminates” persons (or households) at the margins of the income distribution which generates a reduction of income inequality.

3.2 Direct and indirect demographic impacts on the inequality of equivalent incomes

3.2.1 Preliminary remark

With respect to the connections between (socio-)demography and income inequality a differentiation between direct and indirect impacts of (socio-)demography makes sense. Direct demography effects are solely defined by changes in population shares on the supposition of constant economic variables (like mean incomes or dispersions of incomes for the several socio-demographic groups). In contrast, indirect demographic effects aim at economic processes which are relevant for distributional purposes. Examples for such economic processes are individual adjustment reactions (e. g. on the labour market with dependencies on an individual’s own age but also on other market participants’ age) or fiscal aspects like the compliance of budget restrictions.

Simplified (and in a cross-sectional perspective) and referring to the different income sources – labour income, capital income, and (net) transfers –, three main transmission channels for indirect demographic changes exist: the labour market, the capital market, and the tax-transfer system. Additionally, in Figure 4 the channels “application of income” and “macroeconomic level” are depicted. These latter channels are only subordinated influences compared with the first three channels mentioned; partly they depend on the main channels. For reasons of simplicity such interrelations are not pointed out in Figure 4. Moreover, in Figure 4 feedbacks from personal income distribution towards demography (or towards the other indicated variables) are not marked.

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26 See Grimm and Cogneau 2005, pp. 11-12.
27 See von Weizsäcker 1994a, pp. 33-34 (principally, in this context see also von Weizsäcker 1994b).
28 See also in this context Heerink 1994, p. 2.
In the following sections, we will refer to the several transmission channels fragmentarily (and partly exemplarily) to give a short overview.

3.2.2 The direct demographic influence of the household structure

The direct (and differential) demographic effects of fertility and mortality were discussed in Section 3.1 in detail. In this section, additionally, it will be referred to the inequality effects of the household structure.

In this context, Burtless pointed out that changes at the level of households would provide two ways to impact on income inequality. Firstly, the change of a small towards a larger household can lead to the situation that especially young, non-working persons participate in the relatively high incomes of “new” household members. Secondly, gains of welfare result from economies of scale in the context of larger households. If there is no pronounced positive correlation between the corresponding individual incomes, the measured inequality will

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29 The additional question about direct distributional effects of migration can be seen as an “appendix” to the distributional considerations of (differential) fertility and mortality. Furthermore, the socio-demographic variable age can be interpreted as a resultant of the three basic demographic factors mentioned before.

30 Fundamentally, the household structure can be separated into all socio-demographic variables initially mentioned, i. e. age, sex, labour market status, etc. (of the household members, in each case). Since the age distribution depends – as mentioned in the footnote before – on the fundamental demographic factors fertility, mortality, and migration, the household structure itself has some connections to the three other (socio-)demographic variables listed in Figure 4.
be smaller if the adults' individual incomes are pooled within a joint household – compared with the situation in which the corresponding adults live alone. Reversely, that means that inequality will rise if the share of couples decreases and the (positive) correlation between the adults' incomes (in the sense of homogamy) increases. A similar result is reached if low-income receivers live alone disproportionately high.31

Empirically, e. g. in Germany the mean values of the household size have decreased for a long time, as can be seen by Figure 5 for (western) Germany since 1961.

*Figure 5: Mean household sizes in (western) Germany 1961-2008 (arithmetic mean)*

![Figure 5: Mean household sizes in (western) Germany 1961-2008 (arithmetic mean)](http://www.destatis.de)

It is plausible that – ceteris paribus – in case of decreasing mean household sizes the kurtosis of the equivalent income's frequency distribution will be higher in its lower area compared with the status before such a reduction of (mean) household sizes. The reason for this is that the development towards more single-person households reduces the arithmetic mean of the equivalent incomes by tendency. This is the consequence of the fact that a former joint household income of two or more persons within a multi-person household is now disaggregated into two or more incomes for single-person households for which economies of scales are irrelevant. However, opposing effects resulting from a decrease of the mean household sizes can occur with respect to the mean equivalent income: a relatively low fertility can lead to higher (equivalent) incomes for childless couples (for the so-called DINKs; DINK = “double income no kids”).32

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31 See Burtless 2009, pp. 437-441.
32 See in this context Peichl, Pestel, and Schneider 2009, p. 2.
In a number of articles Kuznets (1976, 1981, 1982) stressed the relevance of variations of household size concerning the distribution of household incomes. His arguments focused on a positive correlation between household size and household income. The latter he explained as follows:  

- Larger households would typically consist of more persons in an employable age – with strong incentives to work for these persons because of bigger needs of larger households.
- For larger households, the share of male household heads would be higher than the share of female household heads, and households with male household heads would typically have a higher income than households with female household heads.
- In larger households the share of household heads in a middle-age class would be relatively high, and the middle-aged persons (as household heads) would commonly have higher incomes than young and old persons (as household heads).

Empirically, it was found that a corresponding positive correlation between household size and household income existed in all countries observed, but that the connection between household size and household income per capita was, by tendency, stronger in countries with a rather low mean household size than in countries with a relatively high mean household size.  

The aforementioned aspects primarily referred to mean (equivalent) incomes. With respect to the dispersion of (equivalent) incomes, generated by impacts of household size, it is much more difficult to make substantial points. A more elaborated paper than the one presented here has to deal with this issue.

### 3.2.3 Labour market

In the context of (socio-)demographic influences mediated by the labour market four groups of themes are remarkable: a) demographically caused relative prices of the factor labour, b) the labour market participation of different groups (especially the participation of women), c) group-specific unemployment (rates), and d) the shape and the development of age-earnings profiles.

#### a) Demographically caused relative prices of the factor labour

Here, relative relations of scarceness between the production factors labour and capital (and also within the production factors named) are important; they correspond with wage-interest effects resulting from demographic changes and from changes of the labour force potential. Rodgers stated – on the basis of a (questionable) equalisation of functional and personal income distribution – in a neo-classical sense that a high population's growth would increase the relative supply of the labour force compared with the other production factors; this would

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33 See Kuznets 1981.
34 Qualitatively, this argumentation is also valid concerning the connection between household size and household equivalent income.
35 See Heerink 1994, pp. 196-197. In according with Kuznets' arguments reported above, increases in the shares of women and young persons with respect to all gainfully employed persons lead, furthermore, to a higher degree of earnings' inequality within the group of single-person households and to an increase of earnings' inequality within the group of households with more than one person and with only one income receiver (see Heerink 1994, pp. 199-206).
increase total income inequality (more precisely: ceteris paribus it would reduce the labour incomes versus the incomes stemming from interests; the latter incomes are distributed relatively unequal).  

b) Participation rates of different groups (especially of women) on labour market

Expanding issue a) in this section, the actual group-specific labour force participation is addressed – especially the labour market participation of women. Dudel (2009), e. g., discussed connections between fertility and female labour market participation. On the basis of a model of projection for Germany until 2060, he found a “demographic dilemma”: A higher labour market participation of women would correspond with a lower fertility. Because of this, in the model the German population size will remarkably be reduced until 2060. This would have negative effects upon economic growth in the long run; contrary to this, in the short run the higher labour market participation of women would have positive growth effects. With respect to income inequality, changes of the labour market participation of women would have effects in dependence of the concrete household structures. For example, an increased labour market participation of single mothers would rather reduce than increase income inequality by tendency.

c) Group-specific unemployment (rates)

The question of unemployment exhibits a close link with the question of social inequality: usually the degree of unemployment is positively correlated with (cross-sectional) income inequality (or with relative income poverty). At this, the distributional effects of short-term and long-term unemployment must be separated from each other. Mocan (1999) decomposed – assuming a stochastic trend – the US unemployment rate from 1970 to 1994 into a transitory and into a permanent term. Structural unemployment reduced the income shares of the lower 60 % of the income distribution, and so it caused increases in (income) inequality; it increased (income) inequality stronger than transitory, i. e. short-term unemployment.

d) Age-earnings profiles

Another source of demographically influenced labour market impacts on income distribution are demographically differentiated labour incomes; primarily, this means: age-earnings profiles.  

The empirically well-founded age-earnings profiles can be illustrated by human-capital theory.  

It can be argued that the most investments in human capital (education, on-the-job training, etc.) are typically done in young years of age. These investments require – in the sense of investment costs – reductions of labour incomes in young years of age, whereas the yields of such investments become important in later years of age. This fact leads to a positive connection between age and (labour) income which is reinforced by on-the-job training. Increasing human-capital investments lead to a steeper age-earnings profile. In later years of age depreciations as a consequence of physical and mental “deteriorations” must be considered as a contrast to the sketched increases so that the slope of the age-income profiles declines in these age areas.

The foregoing considerations dealt with age-earnings profiles in a longitudinal perspective. However, age-earnings profiles can also be considered in a cross-sectional way by referring to mean labour incomes of different age groups at a certain point of time – in contrast to the longitudinal perspective where for different cohorts the development of labour incomes over

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36 See in this context Rodgers 1983, p. 443.
37 By the way, age-earnings profiles are only in that sense an element of indirect distributional effects of (socio-)demography that the corresponding profiles reflect age-specific differences in productivity.
38 See in this context the considerations of von Weizsäcker’s multifactorial model mentioned above.
39 See Becker 1964 or Mincer 1974; see also Heerink 1994, pp. 178-179.
time is considered. Because of cyclical and secular trends with respect to the levels of labour incomes or with respect to the factors determining these levels, cross-sectional age-earnings profiles are different from longitudinal profiles.40

Figure 6 shows that in a cross-sectional perspective an inverse U-shaped age-earnings connection can arise despite increasing age-earnings profiles of the several cohorts (the latter means: in a longitudinal perspective). This difference results from longitudinal age-income profiles for the younger cohorts at a higher income level than for the older cohorts.41 Such inverse U-shaped age-related (labour-)income profiles are established in many cross-sectional studies.42

Figure 6: Exemplary age-related (labour-)income profiles in longitudinal and cross-sectional perspective

Source: Author’s own illustration referring to Heerink 1994, p. 179

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40 Heerink 1994, p. 181, provides some examples for cross-sectional age-income profiles. Concerning the generating of typically concave age-income profiles out of different determinants of the age-specific productivity (experience, physique, basic skills like rhetoric, vocabulary, and the like) see e. g. Skirbekk 2008, pp. 194-195. Using US data 1975-1995, Feyrer 2008, pp. 88-90, in addition, showed that patented inventions are also distributed in a concave shape over individual ages (with maxima in the medium age classes between 40 and 50 years).

41 In Figure 6 the youngest cohort is “cohort t”, and the oldest cohort is “cohort t-4”.

42 See in this context e. g. the overview in Kessler and Pestieau 1994, pp. 16-17. See also the very pronounced elucidation of empirical age-earnings profiles for Norway 2000 in Almas, Havnes, and Mogstad 2010, p. 4.
Schröder (2004) expects for Germany in the future that, because of the demographic change, the age-related (labour-)income profiles will be flatter than hitherto (especially in longitudinal perspective but also in a cross-sectional sense). Inter alia he reasons this with relative income losses of the older gainfully employed persons as a consequence of increases in the cohorts’ frequencies of those persons. Additionally, a potential change of age-income profiles might be caused by a (possible further) increase of the pensionable age.\textsuperscript{43}

3.2.4 Capital market

The demographically caused activities on the capital market primarily emerge from the accumulation of wealth over the individual life-cycle and from possible disequilibriums between supply and demand on the capital market.

a) Life-cycle hypothesis

Concerning private, age-dependent (dis-)saving processes and concerning the accumulation of wealth via capital market, in the economic theory typically the life-cycle hypothesis plays an important role (see Figure 7).\textsuperscript{44} It implies that an individual optimises his consumption (of goods and leisure time) over his entire lifetime so that his total utility over lifetime is maximised. In this context, in the literature some typical utility functions are supposed like a time-separable CES utility function where the utility over lifetime depends on period-specific utility values.

According to the life-cycle hypothesis, the private households smooth their consumption through variations of savings, concretely: through dissaving in the phases of education and retirement and through saving in the phase of employment. For example, in Figure 7 the individuals save between their 35\textsuperscript{th} and their 65\textsuperscript{th} year of age; otherwise individual dissaving occurs.\textsuperscript{45}

\textsuperscript{43} See Schröder 2004, pp. 281-282.
\textsuperscript{44} The life-cycle hypothesis stems from Ando and Modigliani 1963 (see also Ando and Modigliani 1964).
\textsuperscript{45} Obviously, in Figure 7 savings are defined as difference between income and consumption. According to the life-cycle hypothesis, total savings equal zero over the entire life-cycle. However, for Germany the life-cycle hypothesis has empirical deficiencies (see Gräf and Schattenberg 2006, pp. 17-19).
b) Asset-meltdown hypothesis and structures on capital market

A further important aspect concerning the connections between demography and capital markets is a reinforced use of capital markets e.g. because of institutional changes from a pay-as-you-go system towards capital covered schemes in the old-age security system. At a certain point of time such changes can lead to an increased number of assets’ sellers compared with a reduced number of potential purchasers. This can cause reductions of assets’ prices and therefore of rates of return on assets (so-called “asset-meltdown hypothesis”).

In the context of an international OLG model, Krueger and Ludwig (2006) stated that the USA probably will “import” the expected, more pronounced demographic changes of the other OECD countries; this can be interpreted as a counter-argument compared to the asset-meltdown hypothesis.\footnote{See also Börsch-Supan 2008, pp. 68-72.} Also in an international view, the (age-dependent) degree of risk aversion and the stage of development of the capital markets are further important determinants of the individual portfolios and, indirectly, of the distribution of capital incomes;\footnote{See Rebeggiani 2007, p. 81.} these latter aspects can also counteract the asset-meltdown hypothesis.
3.2.5 Tax-transfer system

In the context of the theme “Tax-transfer system” – in an exhaustive perspective – it is possible to distinguish between a) public and b) private (net) transfers.

a) Tax system and public transfers

In a comparative-static equilibrium’s model, von Weizsäcker only distinguished between two age groups: gainfully employed persons (in working age) and old-age pensioners. For both age groups income equations exist (in the sense of total net incomes); in this context, the old-age pensioners’ income shall fully equal the retirement benefits (= pensions) which shall be calculated on the basis of a pay-as-you-go system. Referring to the first two moments of a distribution, arithmetic mean and variance, the total income of the whole population can be decomposed into group-specific means and variances. Moreover, von Weizsäcker considered a tax and a public retirement system in his model. The inequality indicator he used was the squared coefficient of variation. Via partial differentiation of this inequality indicator with respect to the relevant variables von Weizsäcker obtained the corresponding inequality influences.48

An important result of this model was that a higher population share of pensioners caused – supposing a balanced national budget – a reduction of (net) income inequality.49 This result referred back to the following assumptions: a) a higher share of gainfully employed persons compared with pensioners, b) a higher mean income of gainfully employed persons in comparison with pensioners, and c) a higher variance of gainfully employed persons’ incomes versus pensioners’ incomes. Moreover, a higher mean age of gainfully employed persons and a higher variance of the working age caused – in distinction from the factor “ratio of pensioners and gainfully employed persons” (see above) – a higher income inequality.50 In this context, von Weizsäcker considered negative incentives for working, as a consequence of increased tax and contribution rates, as well as influences of the age structure on the age-income profile.51 Variations of the demographic parameters in the model’s framework reveal a non-negligible weight of demography concerning measured inequality with an especially high importance of the indirect demographic effects.52 In Chapter 3.3 we will come back to this model.

b) Private intergenerational transfers

The question about private intergenerational transfers – particularly that means: inheritances and donations – is also important in this group of themes (concerning the distribution of wealth or of capital incomes). Whereas e. g. Schloemann (1990, 1992) or Szydlik (1999, 2000) stated an accentuation of wealth inequality via inheritances, Westerheide (2005) argued that inheritances and donations would have levelling effects concerning wealth distribution because of a higher propensity to save, out of the received transfers, in the group of less wealthy households.53

48 See von Weizsäcker 1994a, pp. 35-80.
49 In the context of Krueger and Ludwig’s OLG model above sketched, it can be shown that the “significant” increase of income inequality in their basic model mainly rested upon an increase of the ratio of pensioners and gainfully employed persons; this is a contrary finding to von Weizsäcker’s result. See Krueger and Ludwig 2006, pp. 26-27.
50 See von Weizsäcker 1994a, p. 44 and p. 50.
51 See in this context especially von Weizsäcker 1994a, pp. 61-63.
52 See von Weizsäcker 1994a, p. 76.
53 See also Künemund 2010, pp. 8-10.
3.2.6 Application of incomes

Since an individual income can be either consumed or saved, in the following a) individual consumption and saving patterns as determinants of the personal income inequality will shortly be discussed. Moreover, b) the question about income-dependent needs, as revealed in different equivalence scales over the entire income range, will be touched.

a) Consumption and saving patterns

Schröder discussed the old generation’s expenditure behaviour and its impact on intergenerational transfers as an indirect influence of demographic change upon income distribution and inequality. A (cohort-specifically) altered old generation’s expenditure behaviour towards an enhanced consumption in old years of age can lead to a reduction of the amount of inheritances and donations; this can decrease inequality or, contrary to that, can – on the basis of Westerheide’s (2005) argumentation sketched in Section 3.2.5.b – increase inequality depending on the correlation between inheritances and donations on the one hand and income inequality on the other hand.

b) Income-dependent needs

It seems plausible that the conversion of household incomes into household equivalent incomes requires variable, income-dependent equivalence scales; i. e.: the scale values should be smaller in the upper income area compared with the bottom income classes. The reason for this is that the adding of a further household member – e. g. of a child – only generates low additional costs (in percent) in the upper income area where the reference consumption level (e. g. with respect to housing space) is high. This situation differs from the situation in the bottom income area where a low reference consumption level generates relatively high corresponding additional costs (in percent). Further reasons for income-dependent equivalence scales might be larger price advantages – in the sense of discounts – or better possibilities for indebtedness in the upper compared with the bottom income area.

If corresponding income-dependent equivalence scales are used, the differences of the equivalent incomes between both income areas will rise (ceteris paribus), and so (in a cross-sectional perspective) also (equivalent) income inequality will probably increase.

3.2.7 Macroeconomic level

On the macroeconomic level three central economic variables are important in the current context: a) inflation, b) business activity, and c) economic growth.

a) Inflation

In Mocan’s (1999) analysis – for US data from the 1970s to the mid-1990s – inflation had a progressive influence on the measured income inequality and on its changes over time; reasonably this was only valid for inflation which was not anticipated. A non-anticipated inflation can influence e. g. the distributional relations between gainfully employed persons and pen-

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55 Divergent consumption and saving patterns as consequences of different age compositions in an economy and their impact on the (functional) income distribution and on economic growth were Rada’s (2009) focus in the context of her Keynesian Kaldor model.
56 See in this context Faik 2010, pp. 23-27.
sioners or the relations between the receivers of capital versus labour income. Because of the age-related discrepancies between gainfully employed persons and pensioners and because of lifetime cyclicality of the individual stock of wealth, one obtains cross connections between inflation and demography.

**b) Business activity**

Concerning the connections between business activity and personal income distribution, the above discussed variable unemployment is an important intermediate variable. In a recession, typically, (cyclical) unemployment is higher than in a boom; this increases inequality. However, opposed to the development during booms (when usually profits rise faster than labour incomes), in a recession the ratio of profits and labour incomes is typically lower which points to an inequality decreasing effect of recessions. Until now, dependencies of cyclical effects on demography are not completely clear. In a more elaborated consideration, e. g. it ought to be analysed whether cyclically caused unemployment is rather a phenomenon of young persons or rather one of old persons in an economy.

**c) Economic growth**

With respect to economic growth, Jäntti and Jenkins (2009) stated that – on the basis of the Gini coefficient – for Great Britain a low economic growth would hardly produce any change of inequality, whereas a high economic growth would generate increases of inequality. In some sense this contrasts to “Kuznets’ curve” which postulates a U-shaped connection between the amount of total income per capita and income inequality. The connections between growth and personal income inequality are partly caused by demographic effects. Assuming decreasing factor productivities of the older work force, e. g. a negative connection between demography and growth would be plausible. On such a basis the interplay between growth and income inequality can principally be analysed.

### 3.3 A model embracing the relations between socio-demography and income inequality

Von Weizsäcker (1993) developed a simple model concerning the interplay between demographic change, public finance, individual reactions of adaption, and personal income distribution in an analytically closed form which was already sketched in Section 3.2.5. This model deals with direct as well with indirect (socio-)demographic influences on the personal income distribution. The inequality indicator, von Weizsäcker used, was the squared coefficient of variation $V^2 = \sigma^2/\mu^2$ [$\sigma$: income standard deviation, $\mu$: arithmetic mean value of incomes].

The model's starting point is the segregation of the population into the groups of gainfully employed persons and of pensioners; summing up their incomes, gives the society's income.

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57 See Kuznets 1955.
58 See also Rebeggiani 2007, p. 132.
59 The following considerations refer to von Weizsäcker 1993, pp. 5-87.
For a gainfully employed person j in a specific working period n, the (linear) age-income profile is:

\[ Y_{nj} = A_{nj} - RB_{nj} - SZ_{nj} + TE_{nj} \quad (n = 0, \ldots, N_j) \]

\[ Y_{nj} = A_{nj} - c \cdot A_{nj} - t \cdot (1 - c) \cdot A_{nj} + T \]

\[ = (1 - t) \cdot (1 - c) \cdot A_{nj} + T \quad (n = 0, \ldots, N_j) \]

\[ 0 < t < 1, \quad 0 < c < 1, \quad T > 0 \]

with: \( A_{nj} = a_j + b_j \cdot n \) (\( n = 0, \ldots, N_j ; a_j > 0 ; b_j > 0 \))

\([Y_{nj}]\): net income of the gainfully employed person j in (working) period n, \( A_{nj} \): gross labour income of the gainfully employed person j in (working) period n, \( RB_{nj} \): contribution to the retirement system of the gainfully employed person j in (working) period n, \( SZ_{nj} \): tax payment of the gainfully employed person j in (working) period n, \( TE_{nj} \): transfer income of the gainfully employed person j in (working) period n, \( n = 0 \): entrance into working life, \( n = N_j \): exit out of working life, t: tax rate, c: contribution rate to the retirement system, T: lump-sum transfer, \( a_j = A_{0j} \): initial income, \( b_j = A_{n+1,j} - A_{nj} \): growth of income].

The mean net income of gainfully employed persons' cohort n results in:

\[ \mu_{Y_n} = (1 - t) \cdot (1 - c) \cdot \mu_A + T \quad (n = 0, \ldots, N), \]

where: \( \mu_A = \mu_a + \mu_b \cdot n \).

Thus, the mean net income of all gainfully employed persons \( \mu_Y \) is the sum of mean incomes of all gainfully employed persons' cohorts weighted by the corresponding relative frequencies \( h(n) = J_Y(n)/J_Y \) [\( J_Y(n) \): number of gainfully employed persons in cohort n, \( J_Y \): number of gainfully employed persons]:

\[ \mu_Y = \sum_{n=0}^{N} h(n) \cdot \mu_{Y_n} \]

\[ = (1 - t) \cdot (1 - c) \cdot \mu_A + T, \]

\[ m = \sum_{n=0}^{N} n \cdot h(n) \]

\([m]: \) mean age of gainfully employed persons].

The income of a pensioner i shall be:

\[ P_i = BP_i - SZ_i + TE_i \]

\[ P_i = p \cdot \mu_A \cdot L_i \]

\[ 0 < p < 1, \quad L_i > 0 \]

\([P_i]\): net pension of pensioner i, \( BP_i \): gross pension of pensioner i, \( SZ_i \): tax payments of pensioner i, \( TE_i \): transfers additionally to the gross pension of pensioner i, p: rate of pension payments (controlled by state), \( \mu_A \): mean gross labour income of the gainfully employed persons, \( L_i \): personal assessment basis of pensioner i].
Resulting from a simplification of Equation (18), \( SZ_i \) equals zero; i.e. that the pensions are not taxed. Furthermore and obviously, a dynamisation of pensions is assumed on the basis of the mean gross labour incomes \( \mu_A \).

Considering the mean pension payments \( \mu_P = p \cdot \mu_A \cdot \mu_L \), society’s mean income \( \mu \) can be derived as:

\[
\mu = x \cdot \mu_y + (1 - x) \cdot \mu_p,
\]

(19) \( \text{where: } x = \frac{J_y}{J_y + J_p} = \frac{1}{1 + \frac{J_p}{J_y}} \)

\( J_p \) reflects the number of pensioners, and \( J_p/J_y \) is the ratio of old and young persons or – more precisely: – of pensioners and gainfully employed persons.

Beneath the mean income \( \mu \), the income standard deviation \( \sigma \) (or the income variance \( \sigma^2 \)) is, as is well-known, decisive for the calculation of the squared coefficient of variation \( \sqrt{\sigma^2} \). In this context the income variance of the gainfully employed persons in cohort \( n \) is:

\[
\sigma^2_{\text{y}} = [(1-t)-(1-c)] \cdot \sigma^2_{\text{a}_n} \quad (n = 0, \ldots, N),
\]

(20) \( \text{where: } \sigma^2_{\text{a}_n} = \sigma^2_{\text{a}} + n^2 \cdot \sigma^2_{\text{b}} + 2 \cdot n \cdot \text{cov}(a,b). \)

In the foregoing expression the covariance between \( a \) and \( b \) – with assumed equal values for \( a \) and \( b \) in each cohort – must be considered because the initial income and the growth rate of incomes are not independent of each other: empirically, a negative correlation between both variables was observed in many cases.

On the basis of Equation (20), the income variance of all gainfully employed persons can be decomposed into a within-group element (first term in the first row of Equation (21)) and into a between-group component (second term in the first row of Equation (21)):

\[
\sigma^2_y = \sum_{n=0}^{N} h(n) \cdot \sigma^2_{\text{y}_n} + \sum_{n=0}^{N} h(n) \cdot (\mu_{\text{y}_n} - \mu_y)^2
\]

\[
= \left[(1-t)-(1-c)\right] \cdot \sigma^2_{\text{a}_n},
\]

(21) \( \text{where: } \sigma^2_{\text{a}_n} = \sigma^2_{\text{a}} + (m^2 + s^2) \cdot \sigma^2_{\text{b}} + 2 \cdot m \cdot \text{cov}(a,b) + s^2 \cdot \mu^2_y, \)

\[
s^2 = \sum_{n=0}^{N} (n-m)^2 \cdot h(n)
\]

[\( s^2 \): variance of working age].

Obviously, the stronger the individual initial incomes and the growth rates of income vary and the smaller the (negative) correlation between these two variables is, the larger the income dispersion within the group of gainfully employed persons is. Whereas the life-cycle variable \( b \) is directly connected with age composition, this is not the case for the initial variables \( a \). The reason for this difference can be seen by Equation (21) because the effects of \( \sigma^2_b \) and \( \text{cov}(a,b) \) are, different from the effect of \( \sigma^2_{\text{a}_n} \), influenced by the age structure of gainfully employed persons.

Expression (21) yields, considering the pensioners’ income variance \( \sigma^2_P \) (= \( p^2 \cdot \mu^2_A \cdot \sigma^2_L \)), the societal income variance. In this context, the first two terms in Equation (22) represent the within-group dispersions, and the last term in Equation (22) reflects the between-group dispersion:

\[
\sigma^2 = x \cdot \sigma^2_y + (1-x) \cdot \sigma^2_p + x \cdot (1-x) \cdot (\mu_y - \mu_p)^2.
\]

(22)
In a next step, the tax system is introduced as well as the retirement system (as a pay-as-you-go system); the accounting equations are:

\[
\sum_{n=0}^{N} \sum_{j=1}^{J_{p}(n)} SZ_{nj} + \sum_{i=1}^{N} \sum_{j=1}^{J_{p}(n)} TE_{nj} = \sum_{n=0}^{N} \sum_{j=1}^{J_{p}(n)} T_{nj} \quad \text{(tax system)} \]

\[
\sum_{n=0}^{N} \sum_{j=1}^{J_{p}(n)} t \cdot (1-c) \cdot A_{nj} = \sum_{n=0}^{N} \sum_{j=1}^{J_{p}(n)} T_{nj} \quad \text{(tax system)}^{60}
\]

and

\[
\sum_{n=0}^{N} \sum_{j=1}^{J_{p}(n)} RB_{nj} = \sum_{i=1}^{J_{p}} BP_{i} \quad \text{(retirement system)}.
\]

The contribution rate of the retirement system \(c_{GG}\) equals in the equilibrium:

\[
c_{GG} = \frac{J_{p} \cdot p \cdot \mu_{p} \cdot \mu_{A}}{J_{Y} \cdot x} = \frac{1-x}{x} \cdot p \cdot \mu_{L};
\]

it is the product of the ratio of the old and the young on the one hand and the relative average level of pensions on the other hand. Partial differentiation yields positive dependences of \(c_{GG}\) with respect to \(p\) and \(\mu_{L}\), and a negative dependence on \(x\).

In the equilibrium, the tax rate \(t_{GG}\) is:

\[
t_{GG} = \frac{T}{(1-c_{GG}) \cdot \mu_{A}}
\]

\[
= \frac{T}{\mu_{A} - \frac{J_{p} \cdot p \cdot \mu_{p}}{J_{Y}}}
\]

\[
= \frac{T}{(\mu_{a} + m \cdot \mu_{b}) \cdot \left(1 - \frac{1-x}{x} \cdot p \cdot \mu_{L}\right)}.
\]

The tax rate in the equilibrium depends positively on \(T\), \(p\), and \(\mu_{L}\); moreover, it depends negatively on \(m\), \(x\), \(\mu_{a}\), and \(\mu_{b}\).

In von Weizsäcker’s model an interaction between the contribution rate to the retirement system in the equilibrium and the tax rate in the equilibrium exists, as becomes obvious through the first row of Equation (26). This interaction leads to a dependence of the tax rate in the equilibrium on characteristics of the pensioners (\(J_{p}\), \(p\), and \(\mu_{L}\)) although the pensions are not taxed. The fiscal deductibility of the contributions to the retirement system causes a functional dependence of the minimum income guaranteed \(T\) on the financing of the pensions. If e. g. the number of pensioners \(J_{p}\) increases (or, equivalently, if \(x\) decreases), the contribution rate to the retirement system in the equilibrium \(c_{GG}\) increases, and this reduces the incomes of the gainfully employed persons which are subject to income tax. Thus, the fiscal revenue of the state decreases which requires an increasing tax rate in the equilibrium \(t_{GG}\) (in order to guar-

---

60 As mentioned above, the pensions are not taxed so that the pensioners do not pay taxes.
antee the balance of the public budget). So the tax rate in the equilibrium \( t_{GG} \) rises if \( J_{p} \) increases (and the same holds true for \( p \) and \( \mu_{L} \)).

The (mean) income of the gainfully employed persons, known from Equation (17), must be modified because of the sketched connections between tax and retirement system:

\[
(17a) \quad \mu_{y,GG} = (\mu_{a} + \mu_{b} \cdot m) \cdot \left(1 - \frac{1-x}{x} \cdot p \cdot \mu_{L}\right),
\]

so that the above Equation (19) for the societal income changes to:

\[
(19a) \quad \mu_{GG} = x \cdot (\mu_{a} + m \cdot \mu_{b}).
\]

The (equilibrium’s) income in Equation (19a) is independent of all policy parameters because all charges “come back” to the population.

Equation (21), which corresponds with the gainfully employed persons, must be modified as follows:

\[
(21a) \quad \sigma_{y,GG}^{2} = \left(1 - \frac{T}{\mu_{a} + m \cdot \mu_{b}} - \frac{1-x}{x} \cdot p \cdot \mu_{L}\right)^{2} \cdot \left[\sigma_{a}^{2} + \left(m^{2} + s^{2}\right) \cdot \sigma_{b}^{2} + 2 \cdot m \cdot \text{cov}(a,b) + s^{2} \cdot \mu_{b}^{2}\right]
\]

which means for Equation (23) (and for the entire population):

\[
(23a) \quad \sigma_{GG}^{2} = x \cdot \left(1 - \frac{T}{\mu_{a} + m \cdot \mu_{b}} - \frac{1-x}{x} \cdot p \cdot \mu_{L}\right)^{2} \cdot \left[\sigma_{a}^{2} + \left(m^{2} + s^{2}\right) \cdot \sigma_{b}^{2} + 2 \cdot m \cdot \text{cov}(a,b) + s^{2} \cdot \mu_{b}^{2}\right]
\]

In order to solve the model analytically (to a high extent) von Weizsäcker meets the following three (empirically resilient) assumptions (as already mentioned in Section 3.2.5):

1. \( J_{y} > J_{p} \) (or \( x > \frac{1}{2} \)), i. e.: there are more gainfully employed persons than pensioners;

2. \( \mu_{y} > \mu_{p} \), i. e.: the mean income of the gainfully employed persons is higher than that of the pensioners;

3. \( \sigma_{y}^{2} > \sigma_{p}^{2} \), i. e.: the income dispersion of the gainfully employed persons is higher than that of the pensioners.

Differentiating the squared coefficient of variation with respect to \( x \), yields (in the sense of the total derivative):

\[
\frac{dV_{GG}^{2}}{dx} = \frac{\partial V^{2}}{\partial x} \cdot dV_{GG} + \frac{\partial V^{2}}{\partial t_{GG}} \cdot dt_{GG} + \frac{\partial V^{2}}{\partial c_{GG}} \cdot dc_{GG}
\]

\[
(27) \quad = \left(1 - t_{GG}\right) \cdot \left(1 - c_{GG}\right) \cdot \left[2 \cdot \frac{\mu_{p}}{\mu_{A}} - x \cdot \left(1 - t_{GG}\right) \cdot \left(1 - c_{GG}\right)\right] \cdot \sigma_{A}^{2} - (2 - x) \cdot \sigma_{p}^{2}
\]

\[
+ \left(\mu_{y,GG} - \mu_{p}\right) \cdot \left[3 - 2 \cdot x\right] \cdot \mu_{p} - x \cdot \mu_{A}
\]
In the first row of Equation (27) the first term represents the direct influence of $x$ on the squared coefficient of variation; in von Weizsäcker’s model it is negative. In the second term in the first row of Equation (27) both mathematical factors are negative so that their product is positive. The latter is also valid for the third term in the first row of Equation (27). All in all, some evidence (including empirical evidence) suggests that the influence of $x$ on the squared coefficient of variation is positive. Thus, a higher population share of pensioners reduces income inequality.\(^6\)

It is possible to refine von Weizsäcker’s model (microeconomically) inter alia through the consideration of individual avoiding strategies or reactions concerning governmental activities or through an assumed dependence of the individual income profile on the age structure of the entire population. Qualitatively, such new operationalizations cause no changes compared with the inequality dependences sketched above, as was shown by von Weizsäcker.

In my opinion von Weizsäcker’s approach at least has the following deficiencies:

- the missing of capital incomes,
- an only rudimentary reproduction of the German tax system,
- the confinement of the social security system on a public retirement system,
- the neglecting of the household context,
- the reduced model structure, i.e. the missing microeconomic foundation of the model,
- the comparative-static character of the model,
- missing feedbacks of the distribution and of the accrual of incomes upon demographic variables like e.g. fertility, and
- the operationalization of the age-income profiles as linear functions.

4. Shift-share and decomposition analyses concerning the connection between demography and income inequality

In many cases, comparative-static analyses of incidence concerning the influence of demography on income inequality were accomplished. At this, the population structure of a base year was assumed to be constant, and through the corresponding differences to the inequality values with variable, actual population shares the demographic inequality influence was derived. This simplified, so-called “shift-share approach”\(^6\) implies that changes in the population structure do not affect the degree of inequality within the subgroups and also not the differences in mean incomes between the several subgroups.

An older shift-share analysis for Great Britain stems from Semple (1975). He analysed changes in household composition with respect to gross and net income distribution. Whereas for the Britain gross income distribution 1961-1973 inequality was reduced over time by changes in household composition, concerning net income distribution Semple stated no substantial changes.\(^6\)

Also in 1975 Danziger and Plotnick explored, for the period of time between 1965 and 1972 and on the basis of microeconomic US data of the Survey of Economic Opportunity (SEO)\(^6\)

\(^{61}\) Another differentiation in von Weizsäcker’s model leads to the constellation that a longer mean working time as well as a higher dispersion of working time increase income inequality.

\(^{62}\) Concerning the concept of shift-share analysis see e.g. Dinwiddy and Reed 1977, pp. 115-120.

\(^{63}\) See Semple 1975, p. 102.
from 1966 and the Current Population Survey (CPS) from March 1973, the influence of (twelve) socio-demographic subgroups on the pre-transfer and the post-transfer distribution of incomes. The inequality indicator they used was the Gini coefficient. With respect to the corresponding (socio-)demographic influences on the pre-transfer and the post-transfer distribution of incomes the authors wrote: “Both of these estimates suggest that about one-half of the observed 6.3 percent increase in the Gini coefficient from .4400 in 1965 to .4679 in 1972 cannot be accounted for by demographic change.”

Like Semple (1975) and also for Great Britain on the basis of a shift-share analysis, Dinwiddy and Reed (1977) analysed four factors: 1. changes in the patterns and in the extent of marriages, 2. changes in the population share of old persons, 3. changes in the population shares of full-time pupils and students, and 4. changes in female labour market participation. Whereas the first three factors increased inequality (weakly), the opposite was the case for the fourth factor.

Mookherjee and Shorrocks’ (1982) decomposition approach showed that – for Great Britain 1965-1980 – changes in the age structure were not very relevant for explaining the Britain (gross) income inequality in the period of time mentioned. They concluded – via the indicator mean logarithmic deviation – that the stated increase of the measured inequality was primarily the result of a stronger concavity of the age-income profiles – especially a consequence of increased female working participation rates –, and to a lower extent it depended on changes in inequality within the several age groups.

By another decomposition analysis – again on the basis of the inequality indicator mean logarithmic deviation – Heerink (1994) explored the effects of age and sex on the individual income inequality in a monography which principally dealt with aspects of economic development. He decomposed the mean logarithmic deviation into a between-group and into a within-group component. In the context of economic development the age effect can have two opposite effects on individual income inequality. On the one hand it can become more important if human-capital investments rise; this is positively connected with economic development and probably leads to steeper age-income profiles and to more income inequality. On the other hand the population shares of young population groups typically decrease with a rising degree of economic development so that their weights and at the same time the relative importance of their comparatively low incomes decrease; this reduces inequality. Empirically, it was shown that the absolute amount of the age effect increased with a rising degree of economic development. Furthermore, there was empirical evidence that sex-related income differences were not unimportant concerning a country’s individual income inequality.

Harding (1994) elicited the influences of several socio-demographic variables upon the Australian (gross) income distribution in 1990 by alternatively using the socio-demographic structures of the years 1982 and 1993 as basis for comparisons. She kept the economic variables of influence constant. The inequality indicator she used was the Gini coefficient. In her analysis the personal market income distribution, the personal gross income distribution (i. e. the distribution of market incomes plus transfers), and the personal gross equivalent income distribution were separated from each other. The used equivalence scale was derived out of the Australian literature. As one result, the ageing process observed between 1982 and 1993 only had marginal inequality impacts. In contrast, changes in family structures (towards single-person households and towards couples without children) seemed to have comparatively large inequality increasing effects. Changes in the female labour market participation had inequality reducing effects with respect to the Australian gross equivalent income distribution between 1982 and 1990 (in some contrast to the changes of the male labour market participation rates which only had a very marginal influence on income inequality). Between 1990

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64 Danziger and Plotnick 1975, p. 8.
65 See Mookherjee and Shorrocks 1982, p. 901.
and 1993 there existed, in contrast, no noteworthy inequality effects of changes in the female labour market participation rates in Australia.

Jenkins (1995) showed that changes in the population shares of different household types had no substantial influence on the measured changes of (net equivalent) income inequality in Great Britain between 1971 and 1986. He decomposed the income inequality into different population subgroups on the one hand and into different kinds of income on the other hand, and he referred to the mean logarithmic deviation and the normalised coefficient of variation as decomposable inequality indicators. The population’s breakdown into subgroups revealed a markedly larger influence of within-group inequality compared with between-group inequality.

Karoly and Burtless (1995) analysed changes on labour market and in household composition and their impacts on the income distribution measured by changes of the Gini coefficient in the context of a decomposition approach. Increasing labour market participation rates of female household heads caused an inequality reduction – for the USA in a 30-years period since 1959. The most important demographic factor appeared to be the reduction of household sizes which corresponded with an increase of inequality (especially in the 1970s). However, influences coming from the labour market were quantitatively more important.

Lerman (1996) tried to find out the influences of altered family structures on the poverty of children and on income inequality for the USA 1971-1989. In this context simulation studies showed that the trend towards reduced numbers of marriages was responsible for more than half of increased income inequality. This was caused by two transmission channels: on the one hand by the direct effect of smaller households (with only one or two income receivers) and on the other hand by indirect effects. These latter effects e. g. emerged from larger incentives to work for married men than for non-married men (towards better paid jobs, etc.).

Bishop, Formby, and Smith (1997) compared – on the basis of a regression analysis – the distribution of family incomes in the USA 1976-1989, measured by a socio-demographically adjusted Gini coefficient (analogous to Paglin’s approach sketched at the beginning of the paper), with the corresponding distribution which was measured by a non-adjusted, cross-sectional Gini coefficient. As socio-demographic variables they used age, race, sex, and educational level. They showed that the measured increase of inequality at the end of the period of observation depended less on age and race and more on sex and educational level.

Jäntti (1997) measured – on the basis of cross-sectional LIS data (LIS = Luxembourg Income Study) – the influence of demographic trends on income inequality for several countries; he analysed different age groups within identical household types. Furthermore, Jäntti examined the inequality effects of the number of gainfully employed persons within a household. He concluded that changes in age structure as well as changes in household composition would only have a weak influence on the measured income inequality in countries in which the inequality increases. Another finding was that the inequality within the considered demographic groups (and in a few cases also between those groups) rose. Normally, the growth of the household head’s labour income inequality was the largest “driver” of changes of inequality.

Similarly, Burtless (1999) found that changes in labour income inequality had the strongest impact on changes of the overall income inequality. Pure demographic influences were responsible for approximately one-quarter of the inequality increase observed from 1979 to 1996 in the USA.

Daly and Royer (2000) analysed Californian booms from the end of the 1960s to the end of the 1990s concerning changes of socio-demographic influences on changes of the inequality of household gross equivalent incomes – in comparison with the rest of the USA. By a shift-share analysis they found that socio-demographic factors accounted for between one-third and one-half of the corresponding regional differences.

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68 See Lerman 1996, pp. S119-S120.
An analysis for Australia 1982-1998 made by Johnson and Wilkins (2003) showed that changes in household composition contributed to the measured increases of market income inequality in the amount between one-fifth and one-quarter. The effects of altered household structures were approximately counterbalanced by changes of the Australian age structure, of the population share of foreigners, and of educational levels. Along Jenkins’ (1995) lines, Johnson and Wilkins (2003) also found that changes in the working status accounted for more than half of the measured inequality increase.

In a further shift-share analysis for Australia, concerning the years 1994/95 until 2002/03, Li (2005) stated that changes in all demographic factors together were decisive for approximately one-third of the entire increase of income inequality (in the amount of 2.3 %).

According to the results of Daly and Valletta’s (2006) study, in the USA from 1969 to 1998 demographic influences caused maximally one-quarter of the observed inequality increases. Daly and Valletta used the equivalent household income and in this context Buhmann et al.’s equivalence scale operationalized by an exponent in the amount of the square root of household size. They explicitly noted that in the USA, contrasting to Jenkins’ above stated findings for Great Britain, changes in family structures would have been quite substantial with respect to the observed inequality changes.

Muniz (2008) examined in the context of a decomposition analysis, which was strongly different from the shift-share analyses here otherwise considered, how the population shares and the income shares of the poor, the middle-class, and the rich in Brasilia in 1980, 1991, and 2000 did change and how changes did affect income inequality. His analysis showed that the largest fraction of total income inequality (family income per capita) could be ascribed to between-group variations of the population shares; more than 60 % of total inequality was dedicated to a “class effect”.

Burtless’ (2009) analysis showed that in the USA between 1979 and 2004 demographic changes could explain only a rather small fraction of the measured changes of inequality. 85 % of the observed increases of the Gini coefficient would have occurred even if the demographic structure would have been constant.

In the context of a further decomposition analysis – on the basis of the inequality indicator mean logarithmic deviation and of the German database Sozioökonomisches Panel (SOEP) – Peichl, Pestel, and Schneider (2009) investigated the influence of changes in household structure on the development of the German income inequality.

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69 This operationalization of Buhmann et al.’s equivalence scale has the following appearance: 
\[ m_h = S^\theta \left( \theta \leq \theta \leq 1 \right) \] with \( \theta = 0.5 \) (\( m_h \): equivalence scale value, \( S \): household size, \( \theta \): parameter representing economies of scale; see Buhmann et al. 1988, p. 119).

70 See Daly and Valletta 2006, pp. 76-77.

71 See Daly and Valletta 2006, p. 95.

72 See Burtless 2009, pp. 445-446.
**Excursus: The decomposition of the variation of the mean logarithmic deviation**

The variation of the mean logarithmic deviation (MLD) over time – in the sense of the difference of MLD at the points of time $t+1$ and $t$ – can be decomposed, principally, in four terms (an overbar over a variable or over a mathematical expression denotes the average of values at the points of time $t$ and $t+1$):

$$
\Delta \text{MLD} = MLD_{t+1} - MLD_t
$$

$$
\Rightarrow \Delta \text{MLD} = \sum_{g=1}^{G} w_g \cdot \Delta \text{MLD}_g + \sum_{g=1}^{G} \text{MLD}_g \cdot \Delta w_g - \sum_{g=1}^{G} \ln \left( \frac{\mu_g}{\mu} \right) \cdot \Delta w_g - \sum_{g=1}^{G} w_g \cdot \Delta \ln \left( \frac{\mu_g}{\mu} \right)
$$

$$
\Leftrightarrow \Delta \text{MLD} = \sum_{g=1}^{G} w_g \cdot \Delta \text{MLD}_g \quad \text{(term A)} + \sum_{g=1}^{G} \text{MLD}_g \cdot \Delta w_g \quad \text{(term B)}
$$

$$
- \sum_{g=1}^{G} \left[ \ln \left( \frac{\mu_g}{\mu} \right) \right] \cdot \Delta w_g \quad \text{(term C)} - \sum_{g=1}^{G} \left[ \ln \left( \frac{\mu_g}{\mu} \right) \right] \cdot \Delta \ln \left( \frac{\mu_g}{\mu} \right) \quad \text{(term D)}
$$

[MLD: entire mean logarithmic deviation, $\text{MLD}_g$: mean logarithmic deviation within group of persons $g$, $\mu$: overall arithmetic mean of incomes, $\mu_g$: arithmetic mean of incomes within group of persons $g$, $v_g$: group-specific income share, $w_g$: group-specific population share].

Term A reflects the influence of intertemporal changes with respect to within-group inequality, term B represents changes in within-group inequality which refer to changes of the population shares, term C indicates the influence of changes of the population shares on between-group inequality, and term D characterises relative changes of the mean group-specific incomes and their impact on overall inequality changes.

Summarising the foregoing characterisations,

- term A is an expression for within-group inequality which is generated by different characteristics within the groups (these characteristics must be different from the group-constituting characteristics),
- terms B and C represent the demographic component of inequality change,
- and term D reflects the influence of changes in the distribution of the group-specific mean incomes.

**End of excursus**

Using a corresponding decomposition approach, Peichl, Pestel, and Schneider (2009) empirically stated that the German changes in inequality between 1991 and 2003 were mainly generated by changes within the several population groups. Demographic factors enforced temporary rises of income inequality or decelerated temporary declines of income inequality; they were quite important with respect to the level of inequality, and they were more important for the inequality of household gross equivalent incomes than for the distribution of household net equivalent incomes. This points to the conjecture that the German tax-transfer

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73 See also Faik 2010, pp. 9-10.
74 Basically, see in this context Mookherjee and Shorrocks 1982; see also Rodrigues 1993, p. 9, or Peichl, Pestel, and Schneider 2009, pp. 7-9.
system compensates, at least partly, inequality effects which are caused through altered household structures. Moreover, the inequality increases caused by demographic factors in periods with a rather small change of inequality were very effective. The authors interpreted their findings in such a way “(...) that the demographic changes affect income inequality permanently, whereas other influences only selectively have a large influence through business cycles or other overall economic shocks and push back the demographic components in their relevance only temporarily.”

Last but not least, Almas, Havnes, and Mogstad (2010) dealt with the effects of the "baby boom" in Norway upon the measured (labour) income inequality between 1967 and 2004. They referred to Paglin’s (1975) basic approach insofar as they used an age-adjusted Gini coefficient for measuring inequality. Through this, a slight increase of inequality occurred compared with the "classical" Gini coefficient which is not age-adjusted.

By the way, Almas and Mogstadt (2009) also applied their "new" Gini coefficient to the personal wealth distribution in several countries on the database of the Luxembourg Wealth Study (LWS). Like Almas, Havnes, and Mogstad’s (2010) study above mentioned, an age-adjusted Gini coefficient and, additionally, a multivariate regression’s model were used. On this basis, the age effects were derived supposing constancy of other wealth determinants. Compared with the "classical" Gini coefficient, the age-adjusted Gini coefficient did not produce another ranking of the countries analysed with respect to their inequality of wealth. Despite their preference for age-adjusted reassessments of income and wealth inequality, Almas and Mogstadt concluded that, in this special case concerning wealth, age-adjustments did not play a major role.

It is difficult to compare the findings of the studies above sketched. This is because the data used and, mainly, the operationalizations applied differ strongly from each other. For example, this holds true for the operationalization of the income variables (individual income, household gross income, household net income, or household equivalent income) or for the choice of the inequality indicator (e.g. Gini coefficient versus mean logarithmic deviation versus normalised coefficient of variation). A common result of many studies is, in a qualitative manner, that demographically induced inequality variations are especially effective if the measured inequality variation is comparatively small. In times of peace demographic influences primarily play a role in a long-term perspective. This statement must be restricted insofar as in many studies above discussed merely the direct demographic effects upon income inequality were measured and not the – probably more important – indirect demographic effects.

As a synopsis, the several studies of this chapter are presented in Table 1. The criterions to compare the different studies are the area of investigation (differentiated into region and into years of observation), the operationalization of the variables used (welfare and inequality indicator, and socio-demographic variables), and the results of the several studies.

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76 Peichl, Pestel, and Schneider 2009, p. 21 (author’s own translation from German into English).
77 See Almas and Mogstadt 2009, p. 20.
78 See Burtless 2009, pp. 452-453.
Table 1: Synopsis of (shift-share) studies with respect to the connections between socio-demography and inequality of welfare

<table>
<thead>
<tr>
<th>Study</th>
<th>Region(s)</th>
<th>Years of observation</th>
<th>Welfare variable(s)</th>
<th>Inequality indicator(s)</th>
<th>Socio-demographic variable(s)</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semple (1975)</td>
<td>Great Britain</td>
<td>1961-1973</td>
<td>Household gross and household net income</td>
<td>Gini coefficient</td>
<td>Household structure</td>
<td>Changes of household structures generate reductions of gross income inequality, but they have no large effect on household net income.</td>
</tr>
<tr>
<td>Danziger and Plotnick (1975)</td>
<td>USA</td>
<td>1965, 1972</td>
<td>Gross income</td>
<td>Gini coefficient</td>
<td>12 socio-demographic groups</td>
<td>50 % of the increases of inequality between 1965 and 1972 have socio-demographic causes.</td>
</tr>
<tr>
<td>Mookherjee and Shorrocks (1982)</td>
<td>Great Britain</td>
<td>1965-1980</td>
<td>Gross income</td>
<td>Mean logarithmic deviation</td>
<td>Age structure, female labour market participation</td>
<td>Inequality increases as a consequence of changes in the female labour market participation and, corresponding with this, because of a higher concavity of the age-income profiles. Weaker inequality effects stem from changes in the within-group inequalities of the several age groups.</td>
</tr>
<tr>
<td>Heerink (1994)</td>
<td>Several countries with a different status of economic development</td>
<td>Several years, especially 1970s and 1980s</td>
<td>Individual income (also household gross income)</td>
<td>Mean logarithmic deviation</td>
<td>Age structure, sex, household size</td>
<td>The absolute amount of the age effect – in terms of a steeper age-income profile corresponding with a higher degree of economic development – increases with a higher degree of economic development; the variable sex is relevant for inequality.</td>
</tr>
<tr>
<td>Jenkins (1995)</td>
<td>Great Britain</td>
<td>1971-1986</td>
<td>Household net equivalent income</td>
<td>Mean logarithmic deviation, normalised coefficient of variation</td>
<td>Household type</td>
<td>There is no relevant inequality influence of household types. The within-group inequality is more important than the between-group inequality.</td>
</tr>
</tbody>
</table>
(Table 1 continued:)

<table>
<thead>
<tr>
<th>Study</th>
<th>Region(s)</th>
<th>Years of observation</th>
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<th>Socio-demographic variable(s)</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karoly and Burtless (1995)</td>
<td>USA</td>
<td>30 years since 1959</td>
<td>Household net equivalent income</td>
<td>Gini coefficient</td>
<td>Household size, household composition, sex-specific labour market participation, etc.</td>
<td>The trend towards smaller households increases inequality. Labour market effects are more important for inequality than demographic variables are.</td>
</tr>
<tr>
<td>Lerman (1996)</td>
<td>USA</td>
<td>1971-1989</td>
<td>Household net equivalent income</td>
<td>Gini coefficient</td>
<td>Family structure</td>
<td>Reduced marriage numbers explain around 50% of increased income inequality. This results from smaller households and from stronger incentives to work for married men compared with non-married men.</td>
</tr>
<tr>
<td>Bishop, Formby, and Smith (1997)</td>
<td>USA</td>
<td>1976-1989</td>
<td>Family net income</td>
<td>Original and socio-demographically adjusted Gini coefficient</td>
<td>Age, race, sex, education</td>
<td>The age effect is more important than the race effect concerning income inequality. Furthermore, the impacts of sex and education are more important than the age and the race effect with respect to inequality increases over time.</td>
</tr>
<tr>
<td>Jäntti (1997)</td>
<td>Several countries (LIS database)</td>
<td>1980s</td>
<td>Household net equivalent income</td>
<td>Gini coefficient, mean logarithmic deviation, squared coefficient of variation</td>
<td>Age structure, household type, number of gainfully employed persons per household</td>
<td>In case of a decline of inequality only a small inequality influence of changes in the age and household structure becomes evident. The inequality of labour incomes (of household heads) is an important cause for total income inequality.</td>
</tr>
<tr>
<td>Burtless (1999)</td>
<td>USA</td>
<td>1979-1996</td>
<td>Household net equivalent labour income</td>
<td>Gini coefficient, relations of deciles</td>
<td>Different household types</td>
<td>The inequality of labour incomes is the most important income-related factor concerning total income inequality. Demographic variables account for around ¼ of the observed inequality increase.</td>
</tr>
<tr>
<td>Daly and Royer (2000)</td>
<td>California (USA)</td>
<td>(Mainly) 1969, 1979, 1989 und 1998</td>
<td>Household gross equivalent income</td>
<td>Gini coefficient, relations of deciles</td>
<td>Age, race, education, etc.</td>
<td>Socio-demographic and cyclical effects play a major role concerning the stronger inequality increase in California compared with the rest of the USA. Demographic influences are responsible for one-third up to one-half of the corresponding differences.</td>
</tr>
<tr>
<td>Johnson and Wilkens (2003)</td>
<td>Australia</td>
<td>1982-1998</td>
<td>Market income (on household level)</td>
<td>Gini coefficient, Theil indicator, coefficient of variation, relations of deciles</td>
<td>Household composition, population share of foreigners, age structure, education, working status</td>
<td>Changes of the working status account for more than half of the increase of inequality; changes of household composition constitute a fraction between one-fifth and one-quarter of inequality of market incomes. This effect is approximately counterbalanced by changes of age structure, of population share of foreigners, and of educational levels.</td>
</tr>
<tr>
<td>Li (2005)</td>
<td>Australia</td>
<td>1994/95 versus 2002/03</td>
<td>Household net equivalent income</td>
<td>Gini coefficient</td>
<td>Age structure, sex, working status, place of residence</td>
<td>All demographic factors together account for around one-third of the increase of income inequality (+2.3%).</td>
</tr>
</tbody>
</table>
(Table 1 continued:)

<table>
<thead>
<tr>
<th>Study</th>
<th>Region(s)</th>
<th>Years of observation</th>
<th>Welfare variable(s)</th>
<th>Inequality indicator(s)</th>
<th>Socio-demographic variable(s)</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daly and Valletta (2006)</td>
<td>USA</td>
<td>1969-1998</td>
<td>Household net equivalent income</td>
<td>Gini coefficient, Theil indicator, coefficient of variation, relations of deciles, mean logarithmic deviation</td>
<td>Family structure</td>
<td>Demographic influences are responsible for maximally ¼ of the inequality increase.</td>
</tr>
<tr>
<td>Burtless (2009)</td>
<td>USA</td>
<td>1979-2004</td>
<td>Household net equivalent income</td>
<td>Gini coefficient</td>
<td>Age of household head, household type</td>
<td>Demographic changes only cause a small fraction of inequality changes.</td>
</tr>
<tr>
<td>Peichl, Pestel, and Schneider (2009)</td>
<td>Germany</td>
<td>1991-2003</td>
<td>Household gross equivalent income, household net equivalent income</td>
<td>Mean logarithmic deviation</td>
<td>Household structure</td>
<td>The within-group component is more important than the between-group component. Demography enforces temporary inequality increases (and decelerates temporary inequality reductions). At this, demography is more important for the inequality of household gross equivalent incomes than for the household net equivalent incomes. Above all, demography is most relevant at small changes of inequality.</td>
</tr>
<tr>
<td>Almas and Mogstad (2009)</td>
<td>Several countries (LWS database)</td>
<td>1999-2002</td>
<td>Wealth (per capita)</td>
<td>Original and age-adjusted Gini coefficient</td>
<td>Age structure</td>
<td>There are no noteworthy differences between the modified and the original Gini coefficient.</td>
</tr>
</tbody>
</table>

Source: Author’s own composition

5. Concluding remarks

Demographic variables can influence the personal income inequality directly and indirectly. Indirectly, transmission channels are the labour market, the capital market, the tax-transfer system, the application of incomes, and the macroeconomic level. Former shift-share analyses found in the literature revealed a rather medium influence of demography on measured income inequality (in a cross-sectional perspective). However, in these studies the indirect demographic effects were often excluded. The comprehensive modeling of such indirect demographic effects, e.g. in the context of an OLG model, is a challenge for the future.
Furthermore, longitudinal aspects of the income distribution should be considered in a more exhaustive way as was done so far because in empirical longitudinal studies, beneath an increase of income variance with rising age in the several cohorts, a permanent increase of the average absolute (labour) income with rising age was found.\textsuperscript{79} Now the question is whether lifetime (labour) incomes – i. e. the present values of the individual period (labour) incomes – are distributed more equally or more unequally than the incomes per period.

The previous question cannot be answered terminally because – even in an empirical analysis – such a comparison depends on the income development of the younger cohorts (as Krupp stated). Since the biographies of former cohorts cannot fully serve as an indicator for the income development of younger cohorts (if cohort effects exist), comparisons between lifetime and periodical income distribution need to make suppositions about the further course of incomes for the younger population. Thus, a completed empirical appraisal is not possible.\textsuperscript{80}

Another important and similar aspect concerning the comparison between longitudinal and cross-sectional inequality is the difference between permanent and transitory inequality. In this context Burkhauser and Poupore (1997) stated, on the basis of different income definitions for the confrontation of the USA with western Germany, that both the transitory and the permanent inequality were higher in the USA than in western Germany in the 1980s. At this, in order to determine the permanent, longitudinal income inequality (on an individual level) they used a mobility indicator developed by Shorrocks (R). R is defined as the value of income inequality over a long period of time (i. e., for many years) and a weighted average value of the yearly inequality values. The higher R is the higher the permanent fraction of inequality is. If R equals 1, it is indicated that the economy is fully immobile, and all measured inequality is called permanent.

It is the inclusion of the longitudinal dimension which allows the separation between pure age and cohort effects. Through this, the question about generational equity can be answered more comprehensively than in a pure cross-sectional analysis.\textsuperscript{81}

References


\textsuperscript{80} See Krupp 1983, pp. 177-178.

\textsuperscript{81} For new corresponding longitudinal analyses on the database of the SOEP concerning personal income distribution or concerning the distribution of pensions see e. g. Sopp 2005 or Steiner and Geyer 2010.


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